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ON A SOLUTION OF THE GENERAL BIQUADRATIC EQUATION.

By A. C. BURNHAM, Professor of Mathematics, University of Illinois, Urbana, Illinois.

Very often in mathematical work does one wish to write out without waste of time the value of the unknown in a given biquadratic equation. Nowhere in text-books or mathematical writings do I find the solution to a biquadratic given in such form that one by merely substituting in a formula may get the roots. I have found the formula here given convenient and I do not know that the formula or this particular method of getting the result has ever before been published.

Let the general biquadratic be

$$x^4 + a_1 x^3 + a_2 x^2 + a_3 x + a_4 = 0,$$

and let the roots be a, b, c, d . Then follow, as is well known,

$$\begin{aligned} a+b+c+d &= -a_1, \\ ab+ac+ad+bc+bd+cd &= a_2, \\ abc+abd+acd+bcd &= -a_3, \\ abcd &= a_4. \end{aligned}$$

Now let

$$\left. \begin{array}{l} z_1 = ab + cd \\ z_2 = ac + bd \\ z_3 = ad + bc \end{array} \right\} \dots \dots \dots \text{I.}$$

Then it follows that,

$$\begin{aligned}
 z_1 + z_2 + z_3 &= a_2, \\
 z_1 z_2 + z_1 z_3 + z_2 z_3 &= (ab + cd)(ac + bd) + () + () + () \\
 &= a^2 bc + ab^2 d + c^2 ad + cbd^2 + \dots + \dots \\
 &= \sum a^2 bc = a_1 a_5 - 4a_4,
 \end{aligned}$$

and

$$\begin{aligned} z_1 z_2 z_3 &= (ab + cd)(ac + bd)(ad + be) \\ &= \cancel{\Sigma} a^3 b c d + \cancel{\Sigma} a^2 b^2 c^2 \\ &= a_3^2 + a_1^2 a_4 - 4 a_2 a_4. \end{aligned}$$

Then z_1, z_2, z_3 are therefore the roots of the reducing cubic:

Now from I we have

Therefore by adding (a) and (b),

and by subtracting (a) from (b) we have

In the same manner we get

$$\begin{aligned} \text{But } ab+ac+ad &= a(b+c+d) \\ &= (-a_1 - a)a, \text{ since } b+o+d = -a_1 - a \\ &= -a^2 - a_1 a. \end{aligned}$$

Also $ab + ac + ad = \frac{1}{2}\{z_1 + z_2 + z_3 + 1\sqrt{z_1^2 - 4a_4} + 1\sqrt{z_2^2 - 4a_4} + 1\sqrt{z_3^2 - 4a_4}$
from (c), (e), and (g). Therefore,

$$a^2 + a_1 a + \frac{1}{2} \{ a_2 + \sqrt{z_1^2 - 4a_4} + \sqrt{z_2^2 - 4a_4} + \sqrt{z_3^2 - 4a_4} \} = 0,$$

which is a biquadratic equation giving the value of one root a , *i. e.*

$$a = \frac{-a_1 \pm \sqrt{a_2 - 2\{a_2 + 1\}z_1^2 - 4a_4 + 1\{z_2^2 - 4a_4 + 1\}z_3^2 - 4a_4\}}{2}$$

The four roots are, therefore,

$$\left\{ \begin{array}{l} a \\ b \\ c \end{array} \right\} = \frac{1}{2} \left\{ -a_1 \pm \sqrt{a_1^2 - 2\{a_2 \pm 1, z_1^2 - 4a_4\} \pm 1, z_2^2 - 4a_4 \pm 1, z_3^2 - 4a_4} \right\} \dots \dots \text{III},$$

where the sequence of signs under the main radical, as can be seen from formulae (c) to (h), is

for a , + + +
 for b , + - -
 for c , - + -
 for d , - - +

For the z_1, z_2, z_3 in this solution III must be substituted the roots of the cubic II.

EXAMPLE. As an example take the biquadratic

$$x^4 - x^3 - 7x^2 + x + 6 = 0.$$

Here we have,

$$\begin{aligned}a_1 &= -1, & a_3 &= 1, \\a_2 &= -7, & a_4 &= 6,\end{aligned}$$

from which the cubic becomes $z^3 + 7z^2 - 25z - 175 = 0$, of which the roots are 5, -7, and -5. Thus the roots of the biquadratic are

$$\frac{1}{2}\{1 \pm \sqrt{1-2\{-7 \pm 1 \pm 5 \pm 1\}},$$

or 1, -1, -2, 3, which are seen to be correct.

Care must be exercised that the proper sign before the main radical is taken.

Urbana, Ill., October 9, 1897.

EQUATION OF PAYMENTS.

By J. A. CALDERHEAD, A. B., Professor of Mathematics, Curry University, Pittsburg, Pennsylvania.

Let it be required to find the equated time of two payments, P and P_1 , due at the end of t and t_1 years respectively, and r being the rate of interest.

Represent the equated time by x when $t > t_1$.

I. By SIMPLE INTEREST.

1st Method. The discount on P for $(t-x)$ years must equal the interest on P_1 for $(x-t_1)$ years.

$$\frac{P(t-x)r}{1+(t-x)r} = \text{discount on } P \text{ due } (t-x) \text{ years hence.}$$

$$P_1(x-t_1)r = \text{interest on } P_1 \text{ for } (x-t_1) \text{ years.}$$

$$\therefore \frac{P(t-x)r}{1+(t-x)r} = P_1(x-t_1)r.$$